

Mat 535 Cebir I Ödev 2 (1 Şubat 2016)

1. A ve B , rasyonel sayılar \mathbb{Q} nun toplamsal alt grupları olsun. Eğer $f : A \rightarrow B$ bir grup izomorfizması ise, her $x \in A$ için $f(x) = qx$ olacak şekilde bir $q \in \mathbb{Q}$ olduğunu gösteriniz.
2. Let G, H be finite groups with relatively prime orders. Show that any group homomorphism $f : G \rightarrow H$ is necessarily trivial (that is, sends every element of G to the identity in H .)
3. H abelyan olmak üzere $f : G \rightarrow H$ bir grup homomorfizması veriliyor. Eğer $N, Ker f$ yi içeren G nin bir alt grubu ise N nin normal olduğunu gösteriniz.
4. If G is a finite group, H a subgroup of G such that $|G|$ does not divide $[G : H]!$, prove that there is a normal subgroup $N \neq (e)$ of G contained in H .
5. Prove that a group of order p^2 , p a prime, has a normal subgroup of order p . By using this result prove that group of order p^2 , p a prime, must be abelian.
6. If G is a group and $N \triangleleft G$, show that if $a \in G$ has finite order $|a|$, then Na in G/N has finite order m , where m divides $|a|$.

7.

Show that if H is a cyclic normal subgroup of a finite group G , then every subgroup of H is a normal subgroup of G .

8.

Let G be a group with a normal subgroup N of order 5, such that $G/N \cong S_3$. Show that $|G| = 30$, G has a normal subgroup of order 15, and G has 3 subgroups of order 10 that are not normal.

9.

Let $\sigma = (1\ 2\ 3)(4\ 5\ 6) \in S_6$.

- (a) Determine the size of the conjugacy class of σ and the order of the centralizer of σ in S_6 .
- (b) Determine if $C_{S_6}(\sigma)$ is abelian or non-abelian. Prove your answer.

10.

Let G be a group of order 16 with an element g of order 4. Prove that the subgroup of G generated by g^2 is normal in G .